A Quantitative Study of Optimal Extraction

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1. Introduction

Optimal extraction of spectra from two dimensional formats consists of summing pixels across the spatial profile of the object using weights based on the noise statistics. This can lead to an improvement in the signal-to-noise ratio (S/N) of the extracted one dimensional spectra as compared to not using weights and simply summing the pixels. The statistical theory and some algorithmic implementations of optimal extraction have been described by Robertson (1986), Horne (1986), Marsh (1989), and Kinney, et al. (1990). In these papers there are general statements on the amount of improvement to be expected in certain cases based on either mathematical approximations or examples from real data. The mathematical analysis only gives limiting statements for restricted idealized cases while the examples based on real data only cover a small range of situation. Furthermore, in real data it is difficult to decouple other effects, such as flat field noise or the effects of spectral features, in evaluating the S/N improvements.

In this paper we explore a range of S/N levels and sampling conditions using artificial data, where the true signal and noise properties are well known, to yield quantitative comparisons of the extracted S/N between optimal extraction and simple extraction.

2. The Experiment

A series of artificial, two dimensional long slit spectra of varying signal-to-noise ratio, gaussian profile width, and slopes relative to the image columns are created using the following steps. The true one dimensional spectrum is created with constant flux and no spectral features. The constant, featureless aspect of the model spectra allows the signal-to-noise to be easily and unambiguously determined by a simple mean and standard deviation. The flux levels are increase by factors of 2 to cover the range from low to high S/N in approximately uniform logarithmic intervals. The one dimensional spectrum is convolved with a gaussian profile of specified width and the profile center relative to the image columns varied smoothly with wavelength using a linear trajectory of specified slope. The gaussian profile is then sampled into pixels and a constant sky added. The last step is to add a combination of constant gaussian noise, corresponding

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to detector read out noise, and poissonian photon noise at some detector gain. Since the object of this study is ratios at various signal-to-noise levels the actual signal levels, readout noise, and gain parameters are all scalable and so not important.

The two dimensional spectra are extracted using various aperture widths, sky widths, and algorithms. In each case the spectra are extracted with both optimal weighting and no weighting.

The signal-to-noise ratio for both the weighed and unweighted extractions is computed as the average extracted flux divided by the standard deviation. This simple measure is precise since the true spectrum has a constant value and there are no spectral features or cosmic rays. The artificial spectra have 1000 dispersion points so the precision of the individual measurements is 3%. In addition to the S/N other parameters, such as the actual flux values and the accuracy of the derived uncertainty estimates, are measured but are not reported here.

3. The Software

The study was done entirely with V2.10 IRAF[2]. The artificial data was generated with scripts calling the tasks **mk1dspec**, **mk2dspec**, and **mknoise** in the **artdata** package. The extractions were done with the **apextract** package. The S/N measurments were made with a simple IRAF program written for the project. The IRAF software is freely available and the specific software used in this study may be obtained from the author.

A complete description of the IRAF spectral extraction software is outside the scope and page limits of this paper. Below is a very brief outline of the software. The spectral extraction consists of defining a fixed width apertures along the image axis closest to the spatial axis. The center of the aperture is defined by a smooth function of the dispersion coordinate. Fixed width sky apertures, relative to the center of the aperture, are defined and used for determining the sky in various ways. Sky subtracted aperture spectra are extracted at each point along the dispersion image dimension; extraction is along the image axes even if lines of constant wavelength are not exactly aligned with the axes.

There are three extraction algorithms: simple unweighted summing of the pixels in the aperture with partial pixel weights at the aperture edges, weighted extraction using profile smoothing along image lines or columns parallel to the dispersion axes and a read out and gain model of the noise (called *algorithm 1*), and weighted extraction using a profile smoothing function described by Marsh (called *algorithm 2*). In addition it is possible to use a separate image for defining the profile rather than the data being extracted. While there are differences in some details one may think of algorithm 1 as that described by Horne (1986) and algorithm 2 as that described by Marsh (1989).

[2]Image Reduction and Analysis Facility, distributed by the National Optical Astronomy Observatories

4. The Results

A large number of spectra were extracted with the parameters varied. The number of parameters sets would be extremely large if all parameter space was sampled and, more importantly, it would be very difficult to present and interpret the results. Instead we define one set of parameters to be the standard and then vary individual parameters one at a time. The standard set of parameters have spatial profiles with a full width at half maximum (FWHM) of 3 pixels, a profile center slope relative to the image columns of 1 pixel in 1000 pixels, extraction apertures extending to ± 3 times the gaussian sigma, two sky apertures (one on each side) each of width equal to the object (so that the sampled sky has twice the number of pixels as the object), extraction using algorithm 1 with no pixel rejection, and the profile determined from the data being extracted.

There are a number of quantities which can be compared in this study. In this paper we consider only the S/N of the weighted and unweighted extraction. It is tempting to only look at the S/N improvement, as measured by a ratio or difference. However, then we could not tell whether a S/N improvement is due to a change in the weighted or unweighted extraction. In particular, the ratio could increase either because the weighted S/N increased or the unweighted S/N decreased. For that reason we will look at both the ratio of the weighted to unweighted S/N (or the noise ratio because the extracted fluxes are always nearly identical) and the S/N of the weighted extraction alone. Since we are interested in intercomparisons, the later quantity is normalized by $S/\sqrt{S+1000}$ where S is the true input flux. This factor is an empirical function which approximately normalizes the weighted S/N at all S/N levels.

Figure 1 graphically presents all the results. Each graph is plotted as a function of the extracted weighted S/N on a logarithmic axis. The panels on the left plot the ratio of weighted to unweighted S/N; i.e. the optimal extraction improvement. The panels on the right plot the normalized weighted S/N. The pairs of panels from top to bottom each vary one parameter relative to the standard parameters. The different parameter values are represented by different symbol types. The reference line is the standard model extracted using a high S/N reference image for determining the weighting profile as opposed to determining the profile from the data being extracted.

5. Conclusions

The reader may draw their own conclusions from Figure 1. The conclusions I reach from the data are as follows with supporting panels referenced in brackets. Optimal weighted extraction always improves the S/N, though at high S/N it is insignificant [a-f]. The improvement is maximal around a (weighted) S/N of 2 and actually decreases at lower S/N [a-f]. The peak improvement of about 30% (70% improvement in effective exposure time) for 3σ aperture limits [a-f] agrees well with the expected value derived analytically by Horne (1986).

The improvement in S/N is essentially the same at each S/N level for different spatial profile widths when the extraction aperture is set to the same point in the profile, the 3σ point in panel [a]. However, in this case the narrower profiles achieve a higher actual S/N [g]. This is to be expected since the nar-

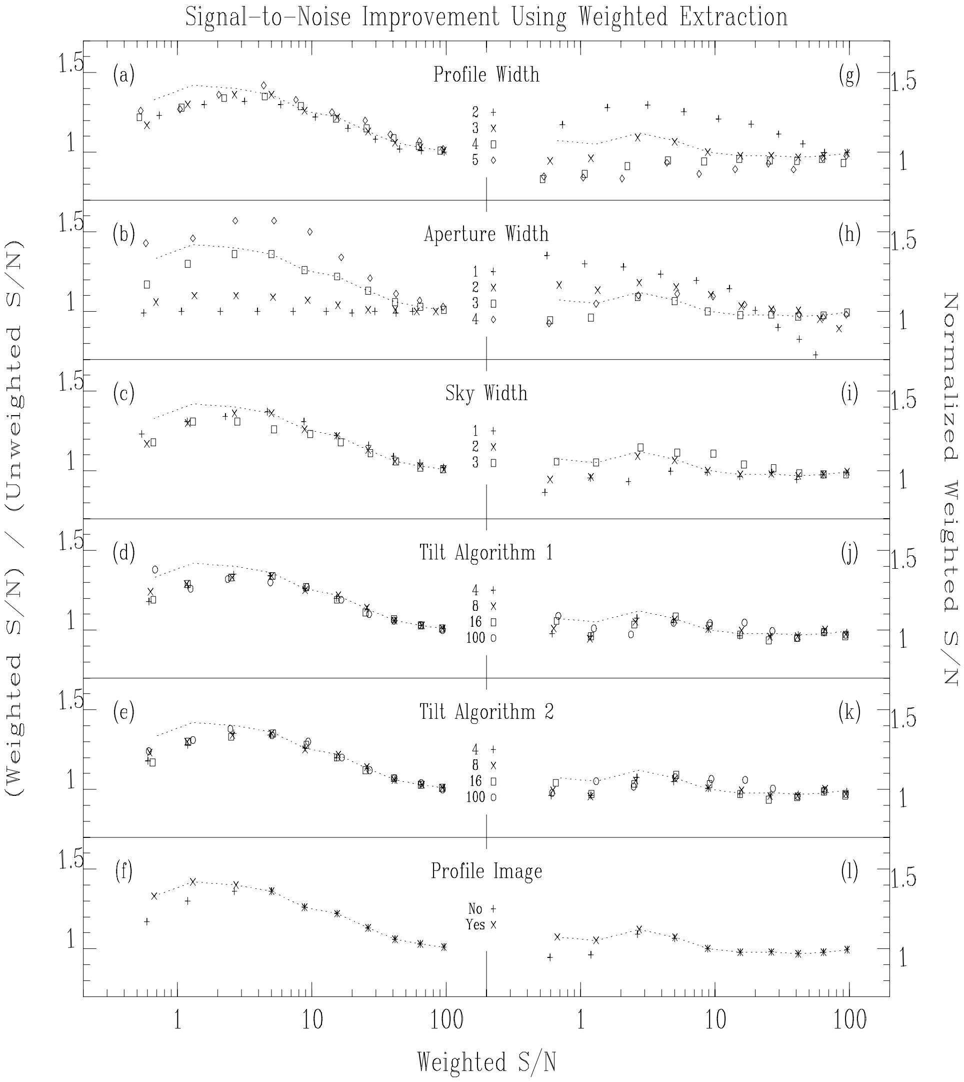

Figure 1. S/N improvements using weighted extraction
A single parameter is varied in each figure relative to the "standard" model and parameters described in the text. The dotted line in all figures is the standard model and paramters with a high S/N image used in determining the weighting profile in place of the profile from the data being extracted. [a,g] Variation of the spectrum profile FWHM in pixels. [b,h] Variation of the aperture width in sigma relative to the profile. [c,i] Variation of the sky width in apertures. [d,j] Variation of the total tilt in pixels from end-to-end using the 1D fitting algorithm. [e,k] Variation of the total tilt in pixels from end-to-end using the Marsh fitting algorithm. [f,l] Comparison of using a separate profile image.

rower profiles put more signal into the central pixels and have a higher contrast relative to the sky.

For a fixed profile width the improvement in S/N appears to increase strongly with larger apertures [b]. However, this is due to the increase in noise in the unweighted extraction as more background noise is added. The actual S/N is higher for narrow apertures when the S/N is low but becomes poorer than wider apertures (for 1 sigma apertures) at S/N greater than about 20 [h]. Thus, for very low S/N, less than 1, very narrow apertures with either weighted or unweighted extractions are best, while at high S/N wider apertures with weighted extraction are indicated. The point that using narrow apertures yields comparable S/N between weighted and unweighted extractions was made by Robertson (1986).

There is little difference in the S/N improvement with the size of the sky apertures [c] but the actual S/N (for low S/N) is improved with wider sky apertures as is expected [i].

There is little difference between using algorithm 1 (based on one dimensional column smoothing) and using algorithm 2 (based on coupled polynomials) and little dependence on slope up to a slope of 100 pixels per 1000 pixels [d,e,j,k]. The good performance of the simpler column fitting is somewhat surprising since the Marsh algorithm was designed for the highly sloped case.

When a high S/N profile can be used for the profile model in low S/N cases, the extraction is improved by a modest amount (14% S/N improvement over the weighted extraction without a profile image at a S/N of 0.6) and becomes negligible at a S/N of 3 [f,l]. Use of a separate profile image is largely, though not exclusively, limited to fiber spectra.

6. Acknowledgements

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References

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